

# Optical Analog-to-Digital Conversion System based on Compressive Sampling

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## Abstract

A novel scheme of optical analog-to-digital conversion system is proposed based on Compressive Sampling and the recently appeared electrical Modulated Wideband Converter. The proposed optical solution is more stable and implementable in practice. Because of the ultra narrow optical pulses width, the proposed solution can provide a uniform Signal-to-Noise attenuate of all frequency bins, compared to its electrical counterpart.

**Keywords:** Compressive Sampling, analog-to-digital conversion, Modulated Wideband Converter, optical sampling.

## 1 Introduction

With the development of radio frequency (RF) technology and digital signal processing, there has been an ever-growing demand for high-rate and high-precision analog-to-digital converters (ADC). Due to the wide spectral that the multiband radio signals may lie in, their Nyquist rates may be far beyond the specifications of state-of-the-art commercial ADCs. Therefore, it has been a challenging task to design a practical AD conversion system that achieves high performances by utilizing commercially available ADCs wisely.

The common approach to reducing the AD sampling rate is to downconvert the narrow-band signal of interest to an intermediate frequency within the bandwidth of the commercial ADC. However, this is based on the fact that the carrier frequency of the signal is known.

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For the case that the carrier information is unavailable, one has to speculate on the worst-case that all possible bands should be demodulated and sampled, and the overall sampling rate turns out to be the Nyquist rate.

To reduce the requirement of ADCs, several approaches are proposed recently [1] [2] [3] using under-sampling and nonuniform sampling. Taking the advantages of ultrashort optical pulses such as low timing jitter and high repetition rates, the sampling scheme in [3] is proposed to be implemented in an optical system. This system is based on under-sampling asynchronously at three different sampling rates. However, as the required sampling rate equals to the repetition rate of optical pulses and can be considerably high, the challenge of high-rate AD conversion still exists.

An innovative system called Modulated Wideband Converter (MWC) [4, 5] is recently proposed based on Compressive Sampling (CS) techniques [6], which makes it possible to acquire adequate information of a sparse signal with limited ADC sampling rate. Instead of sampling all bands separately, the original signal enters several channels simultaneously, while in each channel the signal is multiplied by a periodic waveform to ensure a portion of the energy of all bands appear in the baseband. Then the mixtures are low-pass filtered and sampled at a low rate separately. It is the first possible CS method to break through conventional sampling technologies based on Nyquist-Shannon theory.

Considering the severe demands imposed on the accuracy of amplitude and phase, as well as the stability of sampling pulses, it is difficult to realize a practical and realtime MWC system in electrical circuit. In this letter, the optical implementation of the AD conversion system based on MWC and CS technique is proposed and validated by computer simulations. The characteristic of optical sampling pulses well satisfies the requirement of the proposed structure. Especially, the optical modulation signals composed of narrow pulses can provide a uniform recovery probability and Signal-to-Noise (SNR) attenuate of all frequency bins. Therefore, optical solution can better deal with signals of high frequency compared to the electrical counterpart. Compared to the approach in [3], the proposed system can reduce ADCs' sampling rate to a much lower level and the overall sampling rate of all devices used is smaller than the the Nyquist rate.

## 2 Optical Modulated Wideband Converter

In the scheme of Optical MWC (OMWC), see Fig.1, multiple channels of radio-frequency optical pulses are generated simultaneously by optical pulse sources with different wavelengths. Then these sampling pulses with different wavelengths in separate channels, are modulated respectively by predefined  $T_p$ -periodic sequences that contain  $M$  sampling pulses in each period. After that, all sampling pulses are multiplexed to one channel by an optical multiplexer (MUX) and then enter an Electro-Optic Modulator (EOM).

After passing through a high-pass (HP) filter, the analog signal modulates the sampling pulses in the EOM. The EOM can modulate the amplitude of the optical signal with the

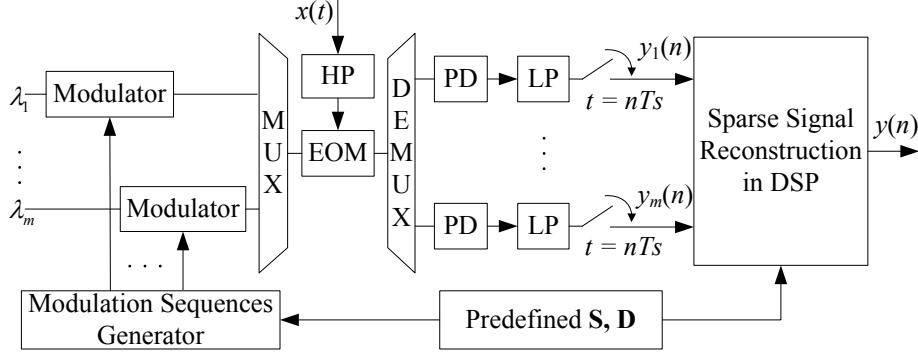


Figure 1: The scheme of Optical Modulated Wideband Converter (OMWC).

electrical signal using a Mach-Zehnder interferometer. Then the modulated sampling pulses are separated by a wavelength demultiplexer (DEMUX) which can split the optical signal into several channels by their wavelengths. Photo Diode (PD) of each channel detects optical signals and then transforms them to electrical analog signals. After passing through the low-pass (LP) filters, electrical analog signals are sampled by parallel electrical ADCs whose sampling rate is the same as the band width of LP filters. Finally, all these digital signals are fed into a sparse signal reconstruction algorithm [5] to recover a Nyquist rate sampling sequence. The recovery algorithm works in two steps [4]. Firstly the support bins, in which narrow band components exist, are calculated. Then the contents inside the support bins are recovered. At the same time, the spectrum of other bins are assigned to zeros.

The principle of OMWC can be explained in the frequency domain. Modulated with periodical sampling pulses, the spectrum of the original signal is shifted by unit of  $1/T_p$ . After passing though the low-pass filter, only the baseband components in each channel are left and to be sampled. To explicate in another way, the spectrum of the original signal is divided into  $L$  bins, shifted to the baseband with respective weights, and then summed together. It is generally recognized that solving  $L$  unknown variables needs  $L$  independent linear equations. However, based on the theory of CS [6], the necessary number of channels for exactly recovering the  $L$  bins components is far less than  $L$ , if most of the  $L$  bins are vacant, i.e., the original spectrum is sparse. Please refer [4] for detailed explanation.

Compared to Electrical MWC (EMWC), there are several advantages in the proposed optics based implementation.

1. Because the optical pulses width is less than one picosecond (ps), their periods can be rather short. Thus we can realize 100GS/s (samples per second) and even faster effective sampling [7]. Besides, the modulation bandwidth can extend into an ultra wide frequency range by using EOM. Considering the powerful ability of DSP, signal processing is accomplished by digital chips. Advantages of optical and electrical technologies are combined to achieve high sampling rate.

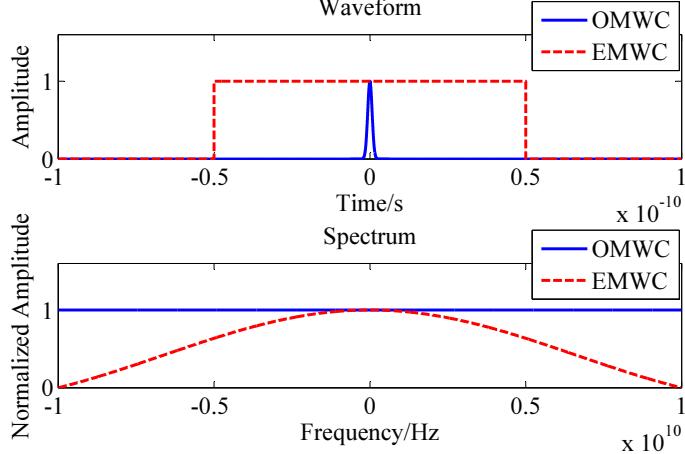


Figure 2: Comparison of optical Gaussian sampling pulse and electrical square sampling pulse in waveform (top) and frequency spectrum (bottom). The spectrum amplitude is normalized.

2. Utilizing the Wavelength Division Multiplexing (WDM) technique [8], the proposed scheme requires only one modulator to sample the signal. This simple and compact setup could reduce the system complexity and, very importantly, avoid the synchronization of multiple EOMs.
3. The jitter of optical pulses produced by mode-locked lasers is less than 1 ps [9]. It is one order of magnitude smaller than electrical sampling pulses and as a result high quantization precision can be achieved using such ultra-stable sampling pulses. Therefore, theoretical result about the spectrum of mixing function is precise enough to reconstruct the signals accurately. It is never needed to calibrate the spectrum of mixing signals in realtime.
4. The  $l$ th spectrum bin of the original signal is modulated to the baseband with the weight  $d_l$ , whose value is closely related to the spectrum of sampling pulses. Considering the narrow width of optical pulse, its spectrum is more flat than that of the electrical square sampling pulse (as depicted in Fig.2). Therefore in OMWC, every bin of the signal spectrum will be regarded with the same weight level when being modulated to the baseband. Thus the desired recovery probability and recovered SNR at all frequency bins are approximately the same.

### 3 Simulation Results and Discussions

In the following two simulations, 40 channels with 10MHz ADCs are used to sample a sparse signal at Nyquist rate 10GHz. Each sampling sequence contains 999 pulses in one period. Consequently the spectrum range of  $-5\text{GHz}$  to  $5\text{GHz}$  is divided into 999 bins. As a result,

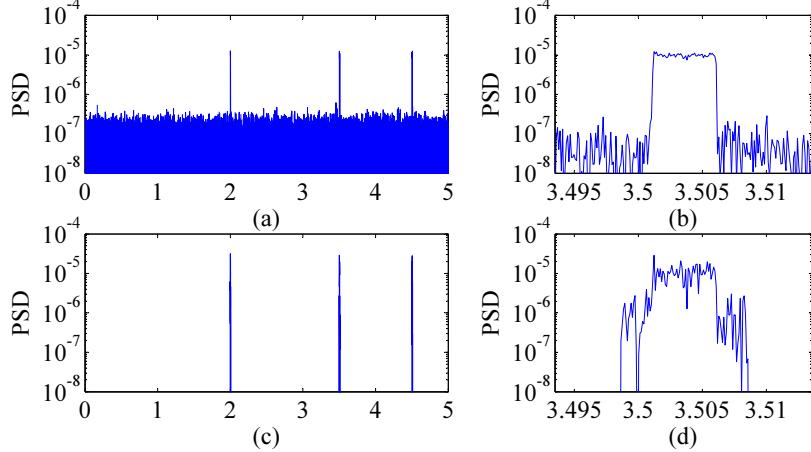


Figure 3: Power spectrum density of (a) original signal and (c) recovered signal, where (b) and (d) are the detail of (a) and (c), respectively. The x-axis denotes frequency in GHz.

the width of each frequency bin is about 10MHz. Optical sampling pulses are set with width 2 ps and period 100 ps.

The first simulation is to demonstrate the basic performance of OMWC. The sparse signal with Gaussian white noise contains 3 narrow band components (5MHz of each and their locations are unknown) and sampled at SNR  $-2.21\text{dB}$  on the whole and  $20\text{dB}$  in their specific bins. The power spectrum density (PSD) of original and recovered signals are depicted in Fig.3. It can be readily accepted that all support bins are correctly identified. Because the spectrum in support bins are recovered using the least square method and that outside the supports are assigned to zeros, one can read from Fig.3(c) that the recovered signal are exactly sparse and the background noise outsides the supports are removed. However, considering the noise from the entire spectrum is down converted to the baseband, the recovered noise in support bins increases. The detail is showed in Fig. 3(b) and (d).

The second simulation is to compare the behaviors of OMWC and EMWC in various scenarios. In the following text, we count bins from the baseband. For example, bin 20 denotes frequency range  $200\text{MHz} \pm 5\text{MHz}$ . Suppose there is a unique narrow band signal of 5MHz, which locates at bin 20, 100, 200, 300, 400, or 490, respectively. The input SNR in its specific bin ranges from  $6\text{dB}$  to  $18\text{dB}$ . In each condition, the simulations are conducted 1000 times and the recovery probability results are plotted in Fig.4. It is readily read that as the SNR increases the recovery probability approaches to 1 and OMWC performs similarly regardless of which bin the signal locates in. Although recovery probability of EMWC is better than OMWC in low frequency, it decreases sharply in high frequency. Furthermore, the improvement of EMWC in low frequency bins is far less than the lost in high ones.

The frequency bin versus average recovery SNR loss is plotted in Fig.5. For the two kinds of MWCs, the spectrum in each frequency bin is modulated to the baseband with various weights (corresponding to the diagonal elements in the matrix  $\mathbf{D}$  in Ref [4]), whose

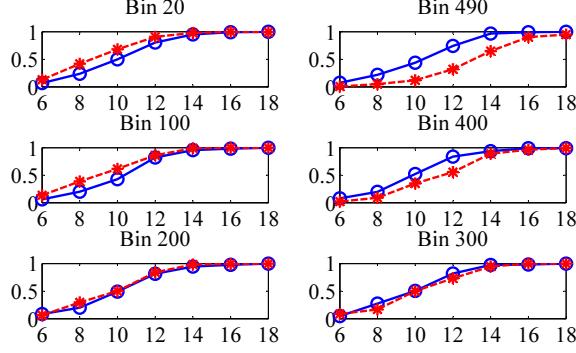


Figure 4: Recovery probabilities of OMWC (blue solid line with circle marker) and EMWC (red dashed line with star marker) in different frequency bins, where x-axis denotes input SNR in the selected band and y-axis denotes recovery probability.

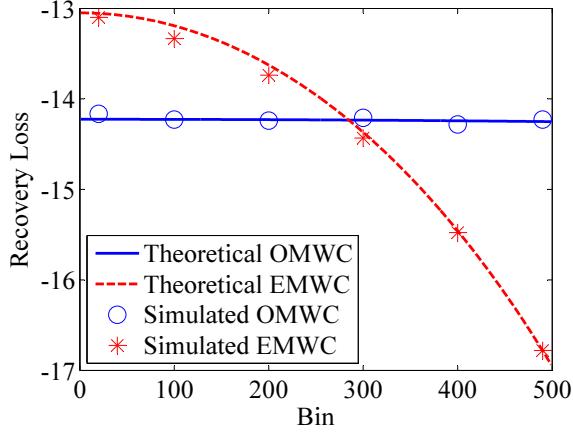


Figure 5: Comparison of recovery SNR loss of OMWC and EMWC in theory and in simulation.

envelop is determined by the spectrum of the sampling pulse. As depicted in Fig.2, the optical pulse width is much narrower than that of electrical one. Therefore, for OMWC, the weights are nearly of the same value. As a result, the recovery loss of OMWC is approximately equal when the carrier frequency of the signal varies. In EMWC, a narrow band signal of low frequency region faces less weighted noise from high frequencies regions than OMWC, because the weights of high frequencies decrease rapidly in EMWC. Consequently, in EMWC there are less average recovery SNR loss in low frequency bins than OMWC. In high frequency regions, the weight of the signal is so small that there are more average recovery SNR loss than OMWC. Recovery SNR of OMWC is at least 2.5dB better than EMWC in high frequency regions while the latter is about 1dB better than OMWC in low frequency bins. Simulation results well demonstrate this effect and the improvement of OMWC over EMWC in high frequency region.

## 4 Conclusion

In this letter a novel optical analog-to-digital conversion system is proposed for sampling sparse wideband signals by using limited channels of off-the-shelf ADCs and compressive sensing techniques. The proposed optical system take advantages of both optical technologies and digital processing methods, achieving highly practical and stable performances compared to its electrical counterpart. The performance is demonstrated by computer simulations.

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